used. If the signal-to-noise ratio in the loop noise bandwidth is 10 dB or greater, the degradation in the theoretical bound is less than 0.5 dB. As the signal-to-noise ratio approaches infinity, the results coincide with a previous analysis by Jordan [3] as would be expected.

It should be noted that these curves apply to a binary input channel with a continuous output. A further degradation for the quantized output channel would occur and would be expected to be of the same order (about 0.2 dB for eight-level quantization) as for the case of a perfect phase reference.

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[1] W. L. Lindsey, "Phase-shift-keyed signal detection with noisy phase reference signals," *IEEE Trans. Aerospace and Electronic Systems*, vol. AES-2, July 1966.
[2] A. J. Viterbi, "Phase-locked loop dynamics in the presence of noise by Fokker-Planck techniques," *Proc. IEEE*, vol. 51, pp. 1737–1753, December 1963.
[3] K. L. Jordan, "The performance of sequential decoding in conjunction with efficient modulation," *IEEE Trans. Communication Technology*, vol. COM-14, June 1966.

[1] A subsidiary of Computer Sciences Corporation.

Comment on "A Useful Recursive Form for Obtaining Inverse z-Transforms"

The result recently presented in the above letter[1] by Jenkins is well known. A brief account of the method is given by Jury,[2] and a tabular way of applying it is given by Pierre.[3]

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[1] L. B. Jenkins, Jr., *Proc. IEEE* (*Letters*), vol. 55, pp. 574–575, April 1967.
[2] E. I. Jury, *Sampled-Data Control Systems*. New York: Wiley, 1958.
[3] D. A. Pierre, "A tabular algorithm for z-transform inversion" (*Data File 74*), *Control Engrg.*, vol. 10, pp. 110–111, September 1963.

Theoretically Unlimited Bandwidth Frequency Divider

***Abstract*—A new method of synthesizing a sine wave frequency divider (division by 2) with theoretically unlimited bandwidth is presented.**

Most methods of sine wave frequency division use tuned circuits or filters such as a digital counter driving a class C amplifier, frequency division by parametric excitation,[1] or regenerative frequency division.[1] All of these methods have limited bandwidth. The new method presented here does not have this drawback.

The mathematics behind the sine wave frequency divider is very simple. Taking the square root of the following trigonometry identity, the magnitude of the half-angle is derived.

$$\cos^2 \theta = \tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta. \tag{1}$$

Let

$$\theta = w_0 t/2 \tag{2}$$

where w_0 is the angular frequency to be divided.

Manuscript received June 12, 1967.
[1] A. A. Kharkevich, *Nonlinear and Parametric Phenomena in Radio Engineering*. New York: Rider, 1962, pp. 176–183.

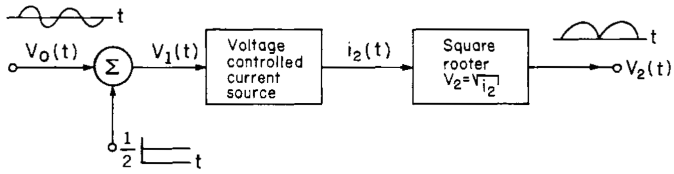

Fig. 1. Block diagram of circuit to synthesize $V_2(t)$ from $V_0(t)$.

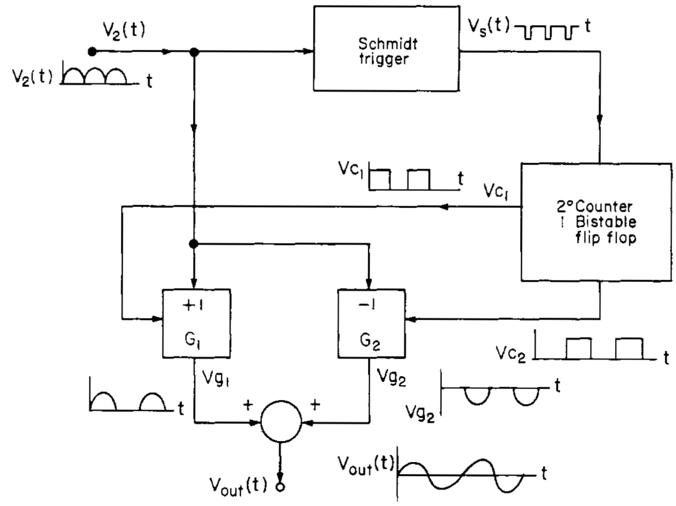

Fig. 2. Block diagram of synchronizer to generate $V_{out}(t)$.

$$\cos^2 w_0 t/2 = \tfrac{1}{2} + \tfrac{1}{2}\cos w_0 t \tag{3}$$

$$|\cos w_0 t| = \sqrt{\tfrac{1}{2} + \tfrac{1}{2}\cos w_0 t}. \tag{4}$$

In order to get the sign as well as the magnitude of the cos $w_0 t/2$, a counting variable $S(|X|)$ is defined which changes sign whenever the magnitude of the X falls below some arbitrary ε.

$$S(X) = \begin{cases} -1, & \text{at the even number of crossings of } |X| < \varepsilon \\ +1, & \text{at the odd number of crossings of } |X| < \varepsilon. \end{cases} \tag{5}$$

Then

$$\cos w_0 t/2 = |\cos w_0 t/2| \cdot S(|\cos w_0 t/2|) \tag{6}$$

$$\cos w_0 t/2 = \sqrt{\tfrac{1}{2} + \tfrac{1}{2}\cos w_0 t} \cdot S\left(\sqrt{\tfrac{1}{2} + \tfrac{1}{2}\cos w_0 t}\right). \tag{7}$$

This expression for the half-frequency is strictly dependent on the fundamental frequency so no phase shift is introduced.

Looking at (7), one can see that the value of $\sqrt{\tfrac{1}{2}+\tfrac{1}{2}\cos w_0 t}$ is the fundamental intermediate variable. If it can be synthesized, then cos $w_0 t/2$ should be able to be synthesized.

In order to simplify notation, define the following variables:

$$V_0 = \tfrac{1}{2}\cos w_0 t \tag{8}$$

$$V_1 = \tfrac{1}{2} + \tfrac{1}{2}\cos w_0 t = \tfrac{1}{2} + V_0(t) \tag{9}$$

$$V_2(t) = |\cos w_0 t/2| = \sqrt{V_1(t)} \tag{10}$$

$$V_{out}(t) = \cos w_0 t/2. \tag{11}$$

Then,

$$V_{out}(t) = V_2(t) \cdot S(V_2(t)). \tag{12}$$

So $V_2(t)$ is the fundamental quantity. One way of synthesizing $V_2(t)$ is shown in Fig. 1.

The sign function $S(V_2(t))$ is a little more difficult to synthesize than is $V_2(t)$ itself. One possible synthesis is shown in Fig. 2. The voltage $V_2(t)$ triggers the Schmidt trigger at the ε crossings. Then the Schmidt trigger output triggers the bistable flip-flop. The two gates G_1 and G_2 are controlled by the bistable flip-flop outputs V_{c_1} and V_{c_2}. G_1 and G_2 are on alternately. The outputs of the gates are V_{g_1} and V_{g_2} which are gated versions of $V_2(t)$ and $-V_2(t)$, respectively. The summer adds V_{g_1} and V_{g_2} to

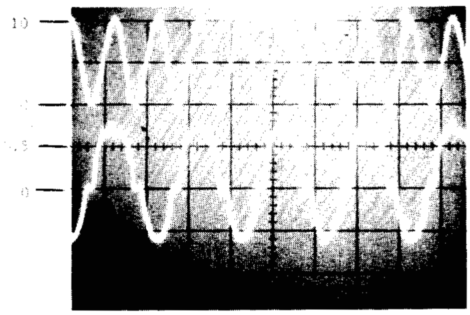

Fig. 3. Upper trace: $V_i(t)$, 5 V/div. Bottom trace: $V_{out}(t)$, 0.5 V/div. Time base: 2 ms/div. Input frequency: 500 Hz.

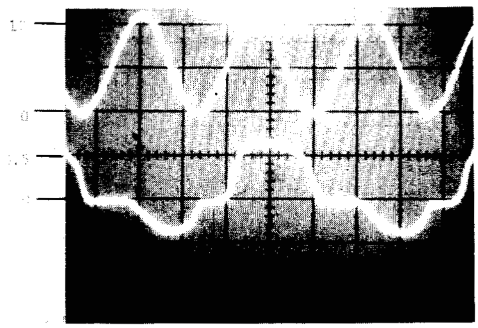

Fig. 4. Upper trace: $V_i(t)$, 5 V/div. Bottom trace: $V_{out}(t)$, 0.5 V/div. Time base: 20 μs/div. Input frequency: 20 kHz.

construct $V_{out}(t)$. With ideal switching and gating the waveform $V_{out}(t)$ should be synthesized exactly.

An experimental design based on this model was built which worked moderately well. The input and output can be seen in Figs. 3 and 4 for an input frequency of 500 Hz and 20 kHz, respectively. The fundamental output component of Fig. 3 is 20 dB above the next highest harmonic. The effective bandwidth of this experimental design was 15 Hz to 20 kHz. Limitations in triggering speeds are the reason for this low bandwidth. However, it is better than the regenerative frequency divider which has a maximum theoretical bandwidth of two-thirds the maximum frequency.

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Comment on "Phase Intercept Distortion in Bandpass Networks"

In regard to the analysis made by Wasylkiwskyj in the above letter,[1] I would like to state the following.

1) The dependence on the phase angle α is a contrived one as the filter has a zero transmission in the vicinity of $\omega=0$ and thus its appearance in the output expression $g(t)$ is artificial. For the "symmetrical" filter [see Fig. 1(a)][2] the significant angle is $\theta(\omega_0)=\alpha_0$.

2) In the case of the symmetrical filter, as implied in the figures of Wasylkiwskyj,[1] the representation in terms of Hilbert transforms is not the clearest way of representing the impulsive response of the filter. Indeed, a simple representation is possible in terms of the low-pass equivalent filter $G_l(\omega)$ [Fig. 1(b)] and its inverse Fourier transform $g_l(t)$. As developed by

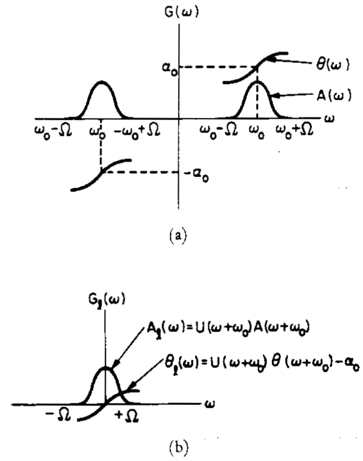

Fig. 1. (a) Symmetrical bandpass filter. (b) Low-pass equivalent filter.

Papoulis,[2] we may write

$$g(t) = 2g_l(t)\cos(\omega_0 t - \alpha_0). \tag{1}$$

When (1) is written in terms of a Hilbert transform we have

$$g(t) = \cos\alpha_0(2g_l(t)\cos\omega_0 t) + \sin\alpha_0\left(\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{2g_l(\tau)\cos\omega_0\tau}{t-\tau}\,d\tau\right), \tag{2}$$

where it is seen that $g_l(t)\cos\omega_0 t$ and $g_l(t)\sin\omega_0 t$ are Hilbert transforms of each other.

3) The distortion implied by the Hilbert transform reduces to a constant phase shift of α_0 in the carrier term. Thus, the only distortion that a modulated low-frequency input to these filters might have would be dependent only on the equivalent low-pass filter's amplitude and phase characteristics.

In the case of the asymmetrical filter, quadrature distortion which is not related to any phase level exists and is treated by Papoulis.[2]

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[1] W. Wasylkiwskyj, *Proc. IEEE (Letters)*, vol. 55, pp. 473–474, March 1967.

[2] A. Papoulis, *The Fourier Transform and Its Applications*. New York: McGraw-Hill, 1965, ch. 7.

Temperature-Gradient Controlled Voltage Tunable Bulk Semiconductor Oscillator

***Abstract*—It was expected that a Gunn oscillator whose electron concentration uniformly decreases from anode to cathode is voltage tunable. Oscillators in which the concentration gradient is maintained by temperature gradient were studied and wideband voltage tunability, similar to that of a tapered oscillator, was observed.**

In the voltage tunable Gunn-effect oscillator reported previously, the frequency control was attained by varying the applied voltage and hence the length of the region where domains travel in a tapered oscillator.[1] A similar control should also be possible in oscillators in which the doping is monotonically reduced from the anode to the cathode. In spite of the simplicity in structure, such a device has not yet been constructed because of the difficulty in introducing a desired doping gradient in GaAs.

In the device described here, we have realized the gradient by introducing a nonuniform temperature distribution. Figure 1 shows the tempera-

Manuscript received May 25, 1967.

[1] M. Shoji, "A voltage tunable Gunn-effect oscillator," *Proc. IEEE (Letters)*, vol. 55, pp. 130–131, January 1967.

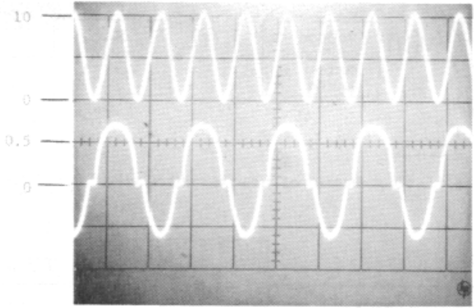

Fig. 3. Upper trace: $V_1(t)$, 5 V/div. Bottom trace: $V_{out}(t)$, 0.5 V/div. Time base: 2 ms/div. Input frequency: 500 Hz.

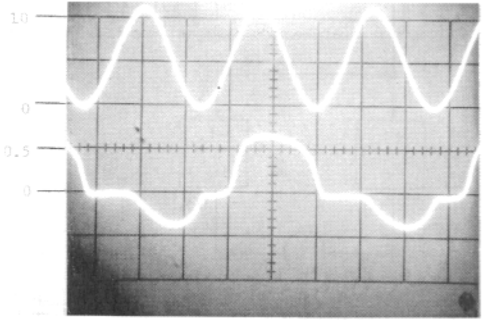

Fig. 4. Upper trace: $V_1(t)$, 5 V/div. Bottom trace: $V_{out}(t)$, 0.5 V/div. Time base: 20 μs/div. Input frequency: 20 kHz.